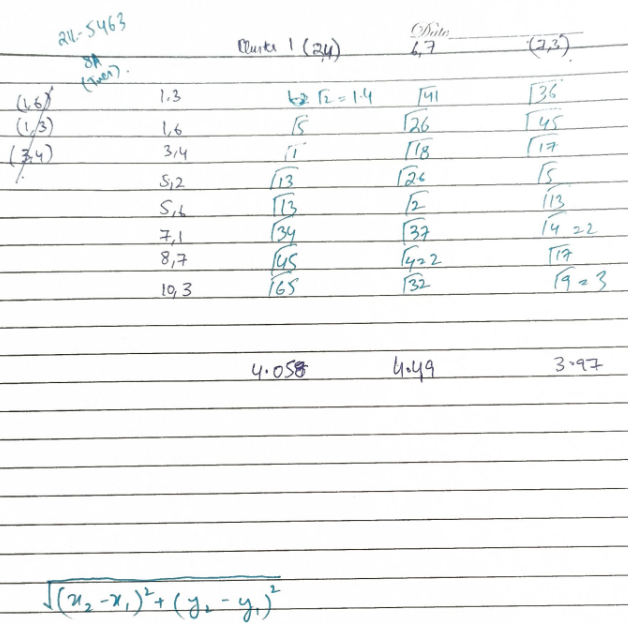
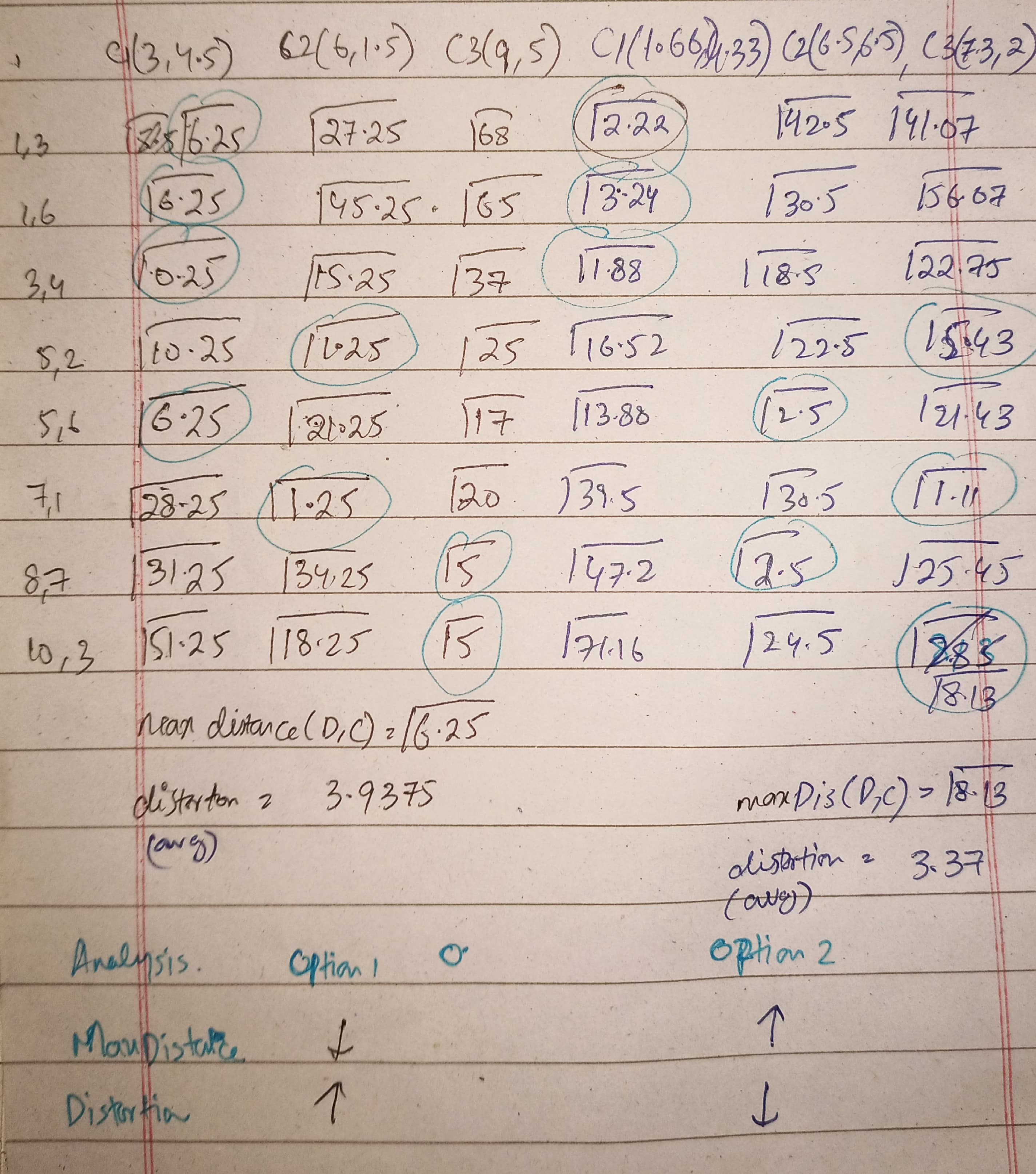
Question 1:

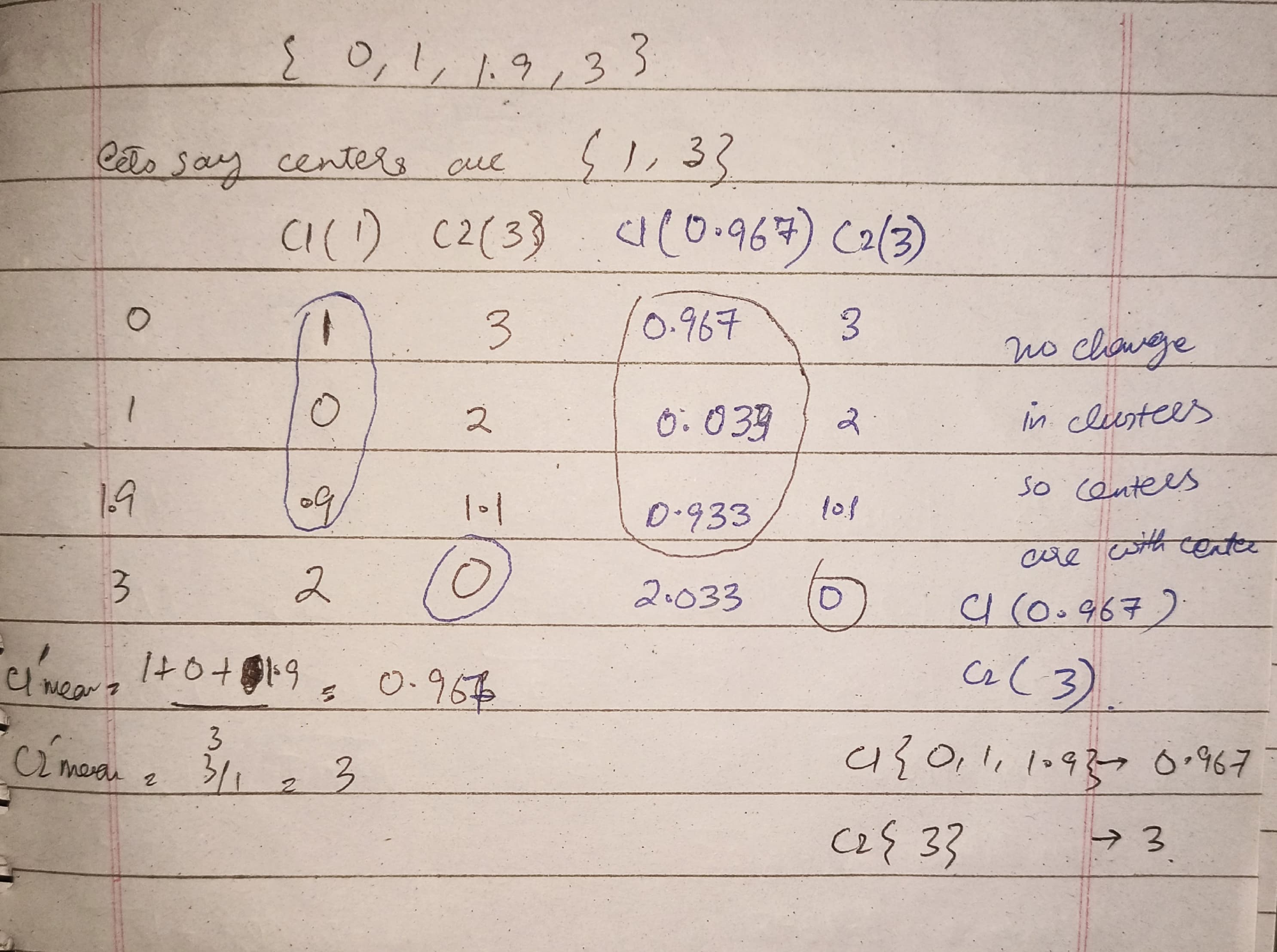


maxDistance (Data, Centers) = 3

Question 2:



Question 3:



Question 4:

**Code**

import numpy as np

def print\_clusters\_and\_matrix(clusters, dist\_matrix, step, merge\_info=None):

    print(f"\nStep {step}:")

    if merge\_info:

        print(f"Merging clusters {merge\_info[0]} and {merge\_info[1]} with distance {merge\_info[2]:.2f}")

    print("\nCurrent clusters:")

    for i, cluster in enumerate(clusters):

        print(f"Cluster {i+1}: {cluster}")

    print("\nDistance matrix:")

    header = "      " + "   ".join([f"C{i+1}" for i in range(len(clusters))])

    print(header)

    for i in range(len(clusters)):

        row = f"C{i+1}  " + "  ".join([f"{dist\_matrix[i,j]:.2f}" for j in range(len(clusters))])

        print(row)

def hierarchical\_clustering(distance\_matrix, labels, linkage='average'):

    # Initialize

    current\_dist = distance\_matrix.copy()

    clusters = [[label] for label in labels]

    step = 0

    print(f"\nInitial distance matrix ({linkage} linkage):")

    print\_clusters\_and\_matrix(clusters, current\_dist, step)

    while len(clusters) > 1:

        step += 1

        # Find the two closest clusters

        min\_dist = np.inf

        for i in range(len(clusters)):

            for j in range(i+1, len(clusters)):

                if current\_dist[i,j] < min\_dist:

                    min\_dist = current\_dist[i,j]

                    merge\_i, merge\_j = i, j

        # Merge the two closest clusters

        new\_cluster = clusters[merge\_i] + clusters[merge\_j]

        merge\_info = (clusters[merge\_i], clusters[merge\_j], min\_dist)

        # Create new distance matrix

        new\_clusters = [clusters[k] for k in range(len(clusters)) if k != merge\_i and k != merge\_j]

        new\_clusters.append(new\_cluster)

        new\_dist = np.zeros((len(new\_clusters), len(new\_clusters)))

        # Fill in distances for existing clusters

        new\_indices = {k: i for i, k in enumerate([idx for idx in range(len(clusters)) if idx != merge\_i and idx != merge\_j])}

        new\_indices[merge\_i] = len(new\_clusters) - 1  # The merged cluster gets the last index

        # Copy unchanged distances

        for i in range(len(clusters)):

            for j in range(len(clusters)):

                if i != merge\_i and i != merge\_j and j != merge\_i and j != merge\_j:

                    new\_i = new\_indices[i]

                    new\_j = new\_indices[j]

                    new\_dist[new\_i, new\_j] = current\_dist[i,j]

        # Calculate new distances based on linkage method

        for k in range(len(clusters)):

            if k != merge\_i and k != merge\_j:

                if linkage == 'average':

                    # Average linkage calculation

                    total = 0

                    count = 0

                    for point\_i in clusters[merge\_i] + clusters[merge\_j]:

                        for point\_j in clusters[k]:

                            idx\_i = labels.index(point\_i)

                            idx\_j = labels.index(point\_j)

                            total += distance\_matrix[idx\_i, idx\_j]

                            count += 1

                    avg\_dist = total / count

                    new\_dist\_val = avg\_dist

                elif linkage == 'single':

                    # Single linkage calculation (minimum distance)

                    min\_d = np.inf

                    for point\_i in clusters[merge\_i] + clusters[merge\_j]:

                        for point\_j in clusters[k]:

                            idx\_i = labels.index(point\_i)

                            idx\_j = labels.index(point\_j)

                            if distance\_matrix[idx\_i, idx\_j] < min\_d:

                                min\_d = distance\_matrix[idx\_i, idx\_j]

                    new\_dist\_val = min\_d

                new\_k = new\_indices[k]

                new\_dist[len(new\_clusters)-1, new\_k] = new\_dist\_val

                new\_dist[new\_k, len(new\_clusters)-1] = new\_dist\_val

        # Print current state

        print\_clusters\_and\_matrix(new\_clusters, new\_dist, step, merge\_info)

        # Update for next iteration

        clusters = new\_clusters

        current\_dist = new\_dist

# Given distance matrix

labels = ['g1', 'g2', 'g3', 'g4', 'g5', 'g6', 'g7', 'g8', 'g9', 'g10']

distance\_matrix = np.array([

    [0.0, 8.1, 9.2, 7.7, 9.3, 2.3, 5.1, 10.2, 6.1, 7.0],

    [8.1, 0.0, 12.0, 0.9, 12.0, 9.5, 10.1, 12.8, 2.0, 1.0],

    [9.2, 12.0, 0.0, 11.2, 0.7, 11.1, 8.1, 1.1, 10.5, 11.5],

    [7.7, 0.9, 11.2, 0.0, 11.2, 9.2, 9.5, 12.0, 1.6, 1.1],

    [9.3, 12.0, 0.7, 11.2, 0.0, 11.2, 8.5, 1.0, 10.6, 11.6],

    [2.3, 9.5, 11.1, 9.2, 11.2, 0.0, 5.6, 12.1, 7.7, 8.5],

    [5.1, 10.1, 8.1, 9.5, 8.5, 5.6, 0.0, 9.1, 8.3, 9.3],

    [10.2, 12.8, 1.1, 12.0, 1.0, 12.1, 9.1, 0.0, 11.4, 12.4],

    [6.1, 2.0, 10.5, 1.6, 10.6, 7.7, 8.3, 11.4, 0.0, 1.1],

    [7.0, 1.0, 11.5, 1.1, 11.6, 8.5, 9.3, 12.4, 1.1, 0.0]

])

# Run both linkage methods

print("="\*80)

print("HIERARCHICAL CLUSTERING WITH AVERAGE LINKAGE")

print("="\*80)

hierarchical\_clustering(distance\_matrix, labels, linkage='average')

print("\n" + "="\*80)

print("HIERARCHICAL CLUSTERING WITH SINGLE LINKAGE")

print("="\*80)

hierarchical\_clustering(distance\_matrix, labels, linkage='single')

**Output**

HIERARCHICAL CLUSTERING WITH AVERAGE LINKAGE

Initial distance matrix (average linkage):

Step 0:

Current clusters:

Cluster 1: ['g1']

Cluster 2: ['g2']

Cluster 3: ['g3']

Cluster 4: ['g4']

Cluster 5: ['g5']

Cluster 6: ['g6']

Cluster 7: ['g7']

Cluster 8: ['g8']

Cluster 9: ['g9']

Cluster 10: ['g10']

Distance matrix:

C1 C2 C3 C4 C5 C6 C7 C8 C9 C10

C1 0.00 8.10 9.20 7.70 9.30 2.30 5.10 10.20 6.10 7.00

C2 8.10 0.00 12.00 0.90 12.00 9.50 10.10 12.80 2.00 1.00

C3 9.20 12.00 0.00 11.20 0.70 11.10 8.10 1.10 10.50 11.50

C4 7.70 0.90 11.20 0.00 11.20 9.20 9.50 12.00 1.60 1.10

C5 9.30 12.00 0.70 11.20 0.00 11.20 8.50 1.00 10.60 11.60

C6 2.30 9.50 11.10 9.20 11.20 0.00 5.60 12.10 7.70 8.50

C7 5.10 10.10 8.10 9.50 8.50 5.60 0.00 9.10 8.30 9.30

C8 10.20 12.80 1.10 12.00 1.00 12.10 9.10 0.00 11.40 12.40

C9 6.10 2.00 10.50 1.60 10.60 7.70 8.30 11.40 0.00 1.10

C10 7.00 1.00 11.50 1.10 11.60 8.50 9.30 12.40 1.10 0.00

Step 1:

Merging clusters ['g3'] and ['g5'] with distance 0.70

Current clusters:

Cluster 1: ['g1']

Cluster 2: ['g2']

Cluster 3: ['g4']

Cluster 4: ['g6']

Cluster 5: ['g7']

Cluster 6: ['g8']

Cluster 7: ['g9']

Cluster 8: ['g10']

Cluster 9: ['g3', 'g5']

Distance matrix:

C1 C2 C3 C4 C5 C6 C7 C8 C9

C1 0.00 8.10 7.70 2.30 5.10 10.20 6.10 7.00 9.25

C2 8.10 0.00 0.90 9.50 10.10 12.80 2.00 1.00 12.00

C3 7.70 0.90 0.00 9.20 9.50 12.00 1.60 1.10 11.20

C4 2.30 9.50 9.20 0.00 5.60 12.10 7.70 8.50 11.15

C5 5.10 10.10 9.50 5.60 0.00 9.10 8.30 9.30 8.30

C6 10.20 12.80 12.00 12.10 9.10 0.00 11.40 12.40 1.05

C7 6.10 2.00 1.60 7.70 8.30 11.40 0.00 1.10 10.55

C8 7.00 1.00 1.10 8.50 9.30 12.40 1.10 0.00 11.55

C9 9.25 12.00 11.20 11.15 8.30 1.05 10.55 11.55 0.00

Step 2:

Merging clusters ['g2'] and ['g4'] with distance 0.90

Current clusters:

Cluster 1: ['g1']

Cluster 2: ['g6']

Cluster 3: ['g7']

Cluster 4: ['g8']

Cluster 5: ['g9']

Cluster 6: ['g10']

Cluster 7: ['g3', 'g5']

Cluster 8: ['g2', 'g4']

Distance matrix:

C1 C2 C3 C4 C5 C6 C7 C8

C1 0.00 2.30 5.10 10.20 6.10 7.00 9.25 7.90

C2 2.30 0.00 5.60 12.10 7.70 8.50 11.15 9.35

C3 5.10 5.60 0.00 9.10 8.30 9.30 8.30 9.80

C4 10.20 12.10 9.10 0.00 11.40 12.40 1.05 12.40

C5 6.10 7.70 8.30 11.40 0.00 1.10 10.55 1.80

C6 7.00 8.50 9.30 12.40 1.10 0.00 11.55 1.05

C7 9.25 11.15 8.30 1.05 10.55 11.55 0.00 11.60

C8 7.90 9.35 9.80 12.40 1.80 1.05 11.60 0.00

Step 3:

Merging clusters ['g8'] and ['g3', 'g5'] with distance 1.05

Current clusters:

Cluster 1: ['g1']

Cluster 2: ['g6']

Cluster 3: ['g7']

Cluster 4: ['g9']

Cluster 5: ['g10']

Cluster 6: ['g2', 'g4']

Cluster 7: ['g8', 'g3', 'g5']

Distance matrix:

C1 C2 C3 C4 C5 C6 C7

C1 0.00 2.30 5.10 6.10 7.00 7.90 9.57

C2 2.30 0.00 5.60 7.70 8.50 9.35 11.47

C3 5.10 5.60 0.00 8.30 9.30 9.80 8.57

C4 6.10 7.70 8.30 0.00 1.10 1.80 10.83

C5 7.00 8.50 9.30 1.10 0.00 1.05 11.83

C6 7.90 9.35 9.80 1.80 1.05 0.00 11.87

C7 9.57 11.47 8.57 10.83 11.83 11.87 0.00

Step 4:

Merging clusters ['g10'] and ['g2', 'g4'] with distance 1.05

Current clusters:

Cluster 1: ['g1']

Cluster 2: ['g6']

Cluster 3: ['g7']

Cluster 4: ['g9']

Cluster 5: ['g8', 'g3', 'g5']

Cluster 6: ['g10', 'g2', 'g4']

Distance matrix:

C1 C2 C3 C4 C5 C6

C1 0.00 2.30 5.10 6.10 9.57 7.60

C2 2.30 0.00 5.60 7.70 11.47 9.07

C3 5.10 5.60 0.00 8.30 8.57 9.63

C4 6.10 7.70 8.30 0.00 10.83 1.57

C5 9.57 11.47 8.57 10.83 0.00 11.86

C6 7.60 9.07 9.63 1.57 11.86 0.00

Step 5:

Merging clusters ['g9'] and ['g10', 'g2', 'g4'] with distance 1.57

Current clusters:

Cluster 1: ['g1']

Cluster 2: ['g6']

Cluster 3: ['g7']

Cluster 4: ['g8', 'g3', 'g5']

Cluster 5: ['g9', 'g10', 'g2', 'g4']

Distance matrix:

C1 C2 C3 C4 C5

C1 0.00 2.30 5.10 9.57 7.22

C2 2.30 0.00 5.60 11.47 8.72

C3 5.10 5.60 0.00 8.57 9.30

C4 9.57 11.47 8.57 0.00 11.60

C5 7.22 8.72 9.30 11.60 0.00

Step 6:

Merging clusters ['g1'] and ['g6'] with distance 2.30

Current clusters:

Cluster 1: ['g7']

Cluster 2: ['g8', 'g3', 'g5']

Cluster 3: ['g9', 'g10', 'g2', 'g4']

Cluster 4: ['g1', 'g6']

Distance matrix:

C1 C2 C3 C4

C1 0.00 8.57 9.30 5.35

C2 8.57 0.00 11.60 10.52

C3 9.30 11.60 0.00 7.97

C4 5.35 10.52 7.97 0.00

Step 7:

Merging clusters ['g7'] and ['g1', 'g6'] with distance 5.35

Current clusters:

Cluster 1: ['g8', 'g3', 'g5']

Cluster 2: ['g9', 'g10', 'g2', 'g4']

Cluster 3: ['g7', 'g1', 'g6']

Distance matrix:

C1 C2 C3

C1 0.00 11.60 9.87

C2 11.60 0.00 8.42

C3 9.87 8.42 0.00

Step 8:

Merging clusters ['g9', 'g10', 'g2', 'g4'] and ['g7', 'g1', 'g6'] with distance 8.42

Current clusters:

Cluster 1: ['g8', 'g3', 'g5']

Cluster 2: ['g9', 'g10', 'g2', 'g4', 'g7', 'g1', 'g6']

Distance matrix:

C1 C2

C1 0.00 10.86

C2 10.86 0.00

Step 9:

Merging clusters ['g8', 'g3', 'g5'] and ['g9', 'g10', 'g2', 'g4', 'g7', 'g1', 'g6'] with distance 10.86

Current clusters:

Cluster 1: ['g8', 'g3', 'g5', 'g9', 'g10', 'g2', 'g4', 'g7', 'g1', 'g6']

Distance matrix:

C1

C1 0.00

================================================================================

HIERARCHICAL CLUSTERING WITH SINGLE LINKAGE

================================================================================

Initial distance matrix (single linkage):

Step 0:

Current clusters:

Cluster 1: ['g1']

Cluster 2: ['g2']

Cluster 3: ['g3']

Cluster 4: ['g4']

Cluster 5: ['g5']

Cluster 6: ['g6']

Cluster 7: ['g7']

Cluster 8: ['g8']

Cluster 9: ['g9']

Cluster 10: ['g10']

Distance matrix:

C1 C2 C3 C4 C5 C6 C7 C8 C9 C10

C1 0.00 8.10 9.20 7.70 9.30 2.30 5.10 10.20 6.10 7.00

C2 8.10 0.00 12.00 0.90 12.00 9.50 10.10 12.80 2.00 1.00

C3 9.20 12.00 0.00 11.20 0.70 11.10 8.10 1.10 10.50 11.50

C4 7.70 0.90 11.20 0.00 11.20 9.20 9.50 12.00 1.60 1.10

C5 9.30 12.00 0.70 11.20 0.00 11.20 8.50 1.00 10.60 11.60

C6 2.30 9.50 11.10 9.20 11.20 0.00 5.60 12.10 7.70 8.50

C7 5.10 10.10 8.10 9.50 8.50 5.60 0.00 9.10 8.30 9.30

C8 10.20 12.80 1.10 12.00 1.00 12.10 9.10 0.00 11.40 12.40

C9 6.10 2.00 10.50 1.60 10.60 7.70 8.30 11.40 0.00 1.10

C10 7.00 1.00 11.50 1.10 11.60 8.50 9.30 12.40 1.10 0.00

Step 1:

Merging clusters ['g3'] and ['g5'] with distance 0.70

Current clusters:

Cluster 1: ['g1']

Cluster 2: ['g2']

Cluster 3: ['g4']

Cluster 4: ['g6']

Cluster 5: ['g7']

Cluster 6: ['g8']

Cluster 7: ['g9']

Cluster 8: ['g10']

Cluster 9: ['g3', 'g5']

Distance matrix:

C1 C2 C3 C4 C5 C6 C7 C8 C9

C1 0.00 8.10 7.70 2.30 5.10 10.20 6.10 7.00 9.20

C2 8.10 0.00 0.90 9.50 10.10 12.80 2.00 1.00 12.00

C3 7.70 0.90 0.00 9.20 9.50 12.00 1.60 1.10 11.20

C4 2.30 9.50 9.20 0.00 5.60 12.10 7.70 8.50 11.10

C5 5.10 10.10 9.50 5.60 0.00 9.10 8.30 9.30 8.10

C6 10.20 12.80 12.00 12.10 9.10 0.00 11.40 12.40 1.00

C7 6.10 2.00 1.60 7.70 8.30 11.40 0.00 1.10 10.50

C8 7.00 1.00 1.10 8.50 9.30 12.40 1.10 0.00 11.50

C9 9.20 12.00 11.20 11.10 8.10 1.00 10.50 11.50 0.00

Step 2:

Merging clusters ['g2'] and ['g4'] with distance 0.90

Current clusters:

Cluster 1: ['g1']

Cluster 2: ['g6']

Cluster 3: ['g7']

Cluster 4: ['g8']

Cluster 5: ['g9']

Cluster 6: ['g10']

Cluster 7: ['g3', 'g5']

Cluster 8: ['g2', 'g4']

Distance matrix:

C1 C2 C3 C4 C5 C6 C7 C8

C1 0.00 2.30 5.10 10.20 6.10 7.00 9.20 7.70

C2 2.30 0.00 5.60 12.10 7.70 8.50 11.10 9.20

C3 5.10 5.60 0.00 9.10 8.30 9.30 8.10 9.50

C4 10.20 12.10 9.10 0.00 11.40 12.40 1.00 12.00

C5 6.10 7.70 8.30 11.40 0.00 1.10 10.50 1.60

C6 7.00 8.50 9.30 12.40 1.10 0.00 11.50 1.00

C7 9.20 11.10 8.10 1.00 10.50 11.50 0.00 11.20

C8 7.70 9.20 9.50 12.00 1.60 1.00 11.20 0.00

Step 3:

Merging clusters ['g8'] and ['g3', 'g5'] with distance 1.00

Current clusters:

Cluster 1: ['g1']

Cluster 2: ['g6']

Cluster 3: ['g7']

Cluster 4: ['g9']

Cluster 5: ['g10']

Cluster 6: ['g2', 'g4']

Cluster 7: ['g8', 'g3', 'g5']

Distance matrix:

C1 C2 C3 C4 C5 C6 C7

C1 0.00 2.30 5.10 6.10 7.00 7.70 9.20

C2 2.30 0.00 5.60 7.70 8.50 9.20 11.10

C3 5.10 5.60 0.00 8.30 9.30 9.50 8.10

C4 6.10 7.70 8.30 0.00 1.10 1.60 10.50

C5 7.00 8.50 9.30 1.10 0.00 1.00 11.50

C6 7.70 9.20 9.50 1.60 1.00 0.00 11.20

C7 9.20 11.10 8.10 10.50 11.50 11.20 0.00

Step 4:

Merging clusters ['g10'] and ['g2', 'g4'] with distance 1.00

Current clusters:

Cluster 1: ['g1']

Cluster 2: ['g6']

Cluster 3: ['g7']

Cluster 4: ['g9']

Cluster 5: ['g8', 'g3', 'g5']

Cluster 6: ['g10', 'g2', 'g4']

Distance matrix:

C1 C2 C3 C4 C5 C6

C1 0.00 2.30 5.10 6.10 9.20 7.00

C2 2.30 0.00 5.60 7.70 11.10 8.50

C3 5.10 5.60 0.00 8.30 8.10 9.30

C4 6.10 7.70 8.30 0.00 10.50 1.10

C5 9.20 11.10 8.10 10.50 0.00 11.20

C6 7.00 8.50 9.30 1.10 11.20 0.00

Step 5:

Merging clusters ['g9'] and ['g10', 'g2', 'g4'] with distance 1.10

Current clusters:

Cluster 1: ['g1']

Cluster 2: ['g6']

Cluster 3: ['g7']

Cluster 4: ['g8', 'g3', 'g5']

Cluster 5: ['g9', 'g10', 'g2', 'g4']

Distance matrix:

C1 C2 C3 C4 C5

C1 0.00 2.30 5.10 9.20 6.10

C2 2.30 0.00 5.60 11.10 7.70

C3 5.10 5.60 0.00 8.10 8.30

C4 9.20 11.10 8.10 0.00 10.50

C5 6.10 7.70 8.30 10.50 0.00

Step 6:

Merging clusters ['g1'] and ['g6'] with distance 2.30

Current clusters:

Cluster 1: ['g7']

Cluster 2: ['g8', 'g3', 'g5']

Cluster 3: ['g9', 'g10', 'g2', 'g4']

Cluster 4: ['g1', 'g6']

Distance matrix:

C1 C2 C3 C4

C1 0.00 8.10 8.30 5.10

C2 8.10 0.00 10.50 9.20

C3 8.30 10.50 0.00 6.10

C4 5.10 9.20 6.10 0.00

Step 7:

Merging clusters ['g7'] and ['g1', 'g6'] with distance 5.10

Current clusters:

Cluster 1: ['g8', 'g3', 'g5']

Cluster 2: ['g9', 'g10', 'g2', 'g4']

Cluster 3: ['g7', 'g1', 'g6']

Distance matrix:

C1 C2 C3

C1 0.00 10.50 8.10

C2 10.50 0.00 6.10

C3 8.10 6.10 0.00

Step 8:

Merging clusters ['g9', 'g10', 'g2', 'g4'] and ['g7', 'g1', 'g6'] with distance 6.10

Current clusters:

Cluster 1: ['g8', 'g3', 'g5']

Cluster 2: ['g9', 'g10', 'g2', 'g4', 'g7', 'g1', 'g6']

Distance matrix:

C1 C2

C1 0.00 8.10

C2 8.10 0.00

Step 9:

Merging clusters ['g8', 'g3', 'g5'] and ['g9', 'g10', 'g2', 'g4', 'g7', 'g1', 'g6'] with distance 8.10

Current clusters:

Cluster 1: ['g8', 'g3', 'g5', 'g9', 'g10', 'g2', 'g4', 'g7', 'g1', 'g6']

Distance matrix:

C1

C1 0.00